CCGPS
Frameworks
Student Edition

Mathematics

Kindergarten Unit Four
Investigating Addition and Subtraction

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"Making Education Work for All Georgians"

Unit 4: Investigating Addition and Subtraction

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CRITICAL AREA

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. Representing, relating, and operating on whole numbers, initially with sets of objects.

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations, such as 5 + 2 = 7 and 7 – 2 = 5. *(Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.)* Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

OVERVIEW

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although CCGPS states, ***"Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required",*** please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **join, add, separate, subtract, and, same amount as, equal, less, more, compose, decompose.**

Number Sense Trajectory –*Putting It All Together*

Trajectory	**Subitizing**	**Comparison**	**Counting**	**One-to-One Correspondence**	**Cardinality**	**Hierarchical Inclusion**	**Number Conservation**
	Being able to visually recognize a quantity of 5 or less.	Being able to compare quantities by identifying which has more and which has less.	Rote procedure of counting. The meaning attached to counting is developed through one-to-one correspondence.	Students can connect one number with one object and then count them with understanding.	Tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set.	Understands numbers are nested inside of each other and that the number grows by one each count. 9 is inside 10 or 10 is the same as 9 + 1.	The total number of objects remains the same when they are rearranged spatially. 5 is 4&1 OR 3&2.

Each concept builds on the previous idea and students should explore and construct concepts in such a sequence

Number Relationships	Spatial Relationship Patterned Set Recognition	One and Two-More or Less	Understanding Anchors	Part-Part-Whole Relationship
	Students can learn to recognize sets of objects in patterned arrangements and tell how many without counting.	Students need to understand the relationship of number as it relates to +/- one or two. Here students should begin to see that 5 is 1 more than 4 and that it is also 2 less than 7.	Students need to see the relationship between numbers and how they relate to 5s and 10s. 3 is 2 away from 5 and 7 away from 10.	Students begin to conceptualize a number as being made up from two or more parts.

Addition and Subtraction Strategies

One/Two More/Less	**Make a Ten**	**Near Doubles**
These facts are a direct application of the One/Two More/ Less than relationships	Use a quantity from one addend to give to another to make a ten then add the remainder.9 + 7 = 10 + 6	Using the doubles anchor and combining it with 1 and 2 more/less.
Facts with Zero Need to be introduced so that students don't overgeneralize that answers to addition are always bigger.	**Doubles** Many times students will use doubles as an anchor when adding and subtracting.	

Developing Number Sense: The Big Picture

This trajectory is designed to show number sense development through the early years of elementary school. Although the graphic organizer flows horizontally left to right, it also aligns vertically. Each concept builds from the previous stage and is the foundation to developing the number sense required of all students.

Developed by Graham Fletcher and adapted from:
Clements& Sarama *Learning and teaching early math: The learning trajectories approach (2009)*
Van de Walle & Lovin *Teaching Student Centered Mathematics (k-3)*

STANDARDS FOR MATHEMATICAL CONTENT

Understand addition as putting together and adding to, and understand subtractions as taking apart and taking from

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

The standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

Students are expected to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

Problem Types

	Result Unknown	**Change Unknown**	**Start Unknown**
Join/Combine	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Separate/ Decompose	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	**("How many more?" version):** Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **("How many fewer?" version):** Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	**(Version with "more"):** Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(Version with "fewer"):** Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	**(Version with "more"):** Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(Version with "fewer"):** Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

\[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

ENDURING UNDERSTANDINGS

- Addition and subtraction problems are placed in four basic categories: *Joining* problems, *Separating* problems, *Part-Part Whole* problems, and *Comparing* problems.
- When elements are added to or *joined* to a set, there are three quantities involved: the starting amount, the change amount, and the resulting amount.
- A *separating* problem involves three quantities; the starting amount, the change amount (the amount being removed), and the resulting amount; however, the starting amount is the largest amount with the change amount being removed which leaves the resulting amount.
- *Part-Part-Whole* problems involve three quantities: two parts that are combined into one whole
- *Compare* problems involve the comparison between two different quantities. The third quantity does not actually exist but is the difference between the two quantities. When one quantity is compared to another, the first quantity is either more than, less than, or equal to the second quantity.
- Problems can be solved in different ways.
- Problems can be modeled using objects, pictures, and words.
- Various combinations of numbers can be used to represent the same quantity.

(see table on previous page for examples)

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How can I compare one quantity to another?
- How can I find the total when I put two quantities together?
- How can I find what is left over when I take one quantity away from another?
- How can I represent problem situations using objects, pictures, and numbers?
- How can I represent problems using objects, pictures, and numbers?
- How can I solve problems using objects, pictures, words, and words?
- How can I use different combinations of numbers to represent the same quantity?
- How can I use models to represent addition and subtraction?
- How can I use models to represent subtraction?
- How can using benchmark numbers help me when adding or subtracting?
- How do you know when your answer makes sense?
- What happens when I decompose a quantity?
- What happens when I join quantities together?
- What happens when sets are joined or separated?
- What happens when some objects are taken away from a set of objects?
- Why is it important that I can build the number combinations for the number 5? 10?

CONCEPTS/SKILLS TO MAINTAIN

Although many students may have attended pre-school prior to entering kindergarten, this is the first year of school for some students. For that reason, no concepts/skills to maintain will be listed at this time. It is expected that teachers will differentiate to accommodate those students that may enter kindergarten with prior knowledge.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for **teacher reference only** and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

- **Combine**
- **Compose**
- **Separate**
- **Decompose**
- **Compare**
- **Quantity**

STRATEGIES FOR TEACHING AND LEARNING

Provide contextual situations for addition and subtraction that relate to the everyday lives of kindergarteners. A variety of situations can be found in children's literature books. Students then model the addition and subtraction using a variety of representations such as drawings, sounds, acting out situations, verbal explanations and numerical expressions. Manipulatives, like two-color counters, clothespins on hangers, connecting cubes and stickers can also be used for modeling these operations. Kindergarten students should see addition and subtraction equations written by the teacher. Although students might struggle at first, teachers should encourage them to try writing the equations. Students' writing of equations in Kindergarten is encouraged, but it is not required.

Create written addition or subtraction problems with sums and differences less than or equal to 10 using the numbers 0 to 10. It is important to use a problem context that is relevant to kindergarteners. After the teacher reads the problem, students choose their own method to model the problem and find a solution. Students discuss their solution strategies while the teacher represents the situation with an equation written under the problem. The equation should be written by listing the numbers and symbols for the unknown quantities in the order that follows the meaning of the situation. The teacher and students should use the words *equal* and *is the same as* interchangeably.

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations. Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As they come to understand the role and meaning of arithmetic operations in number systems, students gain computational fluency, using efficient and accurate methods for computing.

The teacher can use scaffolding to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

COMMON MISCONCEPTIONS

Students may over-generalize the vocabulary in word problems and think that certain words indicate solution strategies that must be used to find an answer. They might think that the word *more* always means to add and the words *take away* or *left* always means to subtract. When students use the words *take away* to refer to subtraction and its symbol, teachers need to repeat students' ideas using the words *minus* or *subtract*. For example, students use addition to solve this Take from/Start Unknown problem: Seth took the 8 stickers he no longer wanted and gave them to Anna. Now Seth has 11 stickers *left*. How many stickers did Seth have to begin with?

If students progress from working with manipulatives to writing numerical expressions and equations, they skip using pictorial thinking. Students will then be more likely to use finger counting and rote memorization for work with addition and subtraction. Counting forward builds to the concept of addition while counting back leads to the concept of subtraction. However, counting is an inefficient strategy. Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematics.

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Represent the combining of two sets
- Model and understand the concept of part-part whole addition
- Represent the difference between two sets
- Model problem situations using objects, pictures, words and numbers
- Represent number combinations up to 10
- Count one-to-one in counting order to 20
- Understand and model number relationships
- Identify more, less and equal to when comparing sets

- Model multiple representations of the same number
- Solve addition and subtraction word problems within 10
- Decompose numbers less than or equal to 10
- Understand the relationship 0-10 number relationships

TASKS

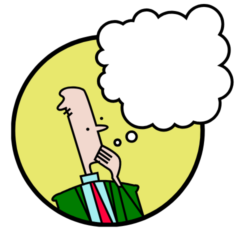
The following tasks represent the level of depth, rigor, and complexity expected of all Kindergarteners. These tasks or a task of similar depth and rigor should be used to demonstrate evidence of learning.

Scaffolding Task	Constructing Task	Practice Task	Performance Task
Tasks that build up to the constructing task.	Constructing understanding through deep/rich contextualized problem solving tasks	Games/activities	Summative assessment for the unit

Task Name	Task Type/ Grouping Strategy	Content Addressed
Addition/Subtraction Word Problems	Constructing Task *Individual. Whole, Small Group*	Addition and subtraction through word problems
Building Trains	Constructing Task *Whole or Small Group*	Number combinations to 5 and 10
Going Bananas	Practice Task *Whole, Small Group or Individual*	Number combinations to 5 through problem solving
Tug-O-War	Practice Task *Partners*	Using a number line to find number combinations to 20
Race to 20 Revisited	Practice Task *Partners*	Number combinations to 5 (+/-)
Shake and Spill Addition/Subtraction	Constructing Task *Whole, Small Group or Individual*	Number combinations to 10 (+/-)
Summoning 5's and 10's	Practice Task *Partners*	Using 5 as a benchmark number, number combinations to 10
Bo Peep's Domino Dilemma	Practice Task *Whole, Small Group or Individual*	Number combinations to 5 and 10
The Candy Store	Practice Task *Individual*	Number combinations to 5 through problem solving
Calling Out Reds? (5/10) Addition/Subtraction	Constructing Task *Whole, Small Group or Individual*	Mental computation of number combinations to 5 or 10.
Dropping Pennies	Performance Task *Individual*	Number combinations to 5 through problem solving
Make 5/10 Go Fish	Practice Task *Partners*	Mental computation of number combinations to 5 or 10.

Farmer McDonald	Practice Task *Whole, Small Group or Individual*	Number combinations to 5 through problem solving
4 Sums in a Row	Practice Task *Partners*	Mental computation of number combinations to 5 or 10.
The Bike Store	Performance Task *Individual*	Number combinations to 5 through problem solving

As this unit has no Culminating Task, you may pair/modify tasks to include all unit standards in combination.



<u>CONSTRUCTING TASK:</u> Addition and Subtraction Word Problems

This task can be modified and used throughout the unit

This task is intended to serve as a model for future tasks that you should create. "*When children are learning about the operations of addition and subtraction, it's helpful for them to see the connection between these processes and the world around them. Word problems accustom children to looking at groups of people or objects, help them see the actions of joining and separating, and give them experience figuring out sums and differences*." (Burns 2006)

STANDARDS FOR MATHEMATICAL CONTENT

Understand addition as putting together and adding to, and understand subtractions as taking apart and taking from

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

As you tell the addition or subtraction stories, have children act out the stories using a variety of manipulatives to represent the people, animals, or objects. Once children are comfortable acting out the stories and demonstrate proficiency with acting them out, show students how to record matching number sentences. The minus sign should be read as "minus" or "subtract" **but not as "take away"**. The plus sign is easier since it is typically a substitute for "and" (Van de Walle, 2010).

As you create word problems for your students, base them on people, animals, or objects that you are discussing in other content areas or that the students can connect with for other reasons. Learning is tied to emotion and if students can connect with the problem, they are more likely to be engaged and therefore, learn through their experience.

This task is designed to serve as an example of how you should recreate this type of task frequently throughout the unit and the remainder of the school year with a variety of books. Since there are several word problem types and you want to allow time for students to explain their thinking, this type of task can be spread across a few math segments to allow for rich mathematical discussion about each problem type.

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- Sample word problem sheet to use as a guide
- *If You Give a Mouse a Cookie by Laura Numeroff* or any book that can be used to create word problems like the ones in the task chart
- Cookie shaped cereal for each student (or if using a different text, a variety of manipulatives for students to choose from)

GROUPING

Whole, Individual, Small group

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION:

Read the story, *If You Give a Mouse a Cookie (*or any book you choose) to students. Give students individual bags of the cookie shaped cereal to use as a math tool for this task. Explain to the students that you will work together to solve Mouse's problem (you may choose to have students work in pairs or small groups during other math segments and later individually once they become more proficient with problem solving in a variety of situations.) Discuss each problem situation on the chart and the various strategies used to find the sum (doubles, finding five or ten, drawing pictures, acting out the story with manipulatives, etc.).

Select word problems from the guide sheet and change them as you see necessary to reflect the concepts you are teaching in other academic areas. Remember, they should only include numbers up to 10 or 12. You could print a few to make a recording sheet so that students can represent their thinking on paper. You should make sure to expose students to a variety of structures (*join* problems, *separate* problems, *part-part-whole* problems, and *compare* problems) regularly so that they can develop proficiency with this type of problem-solving.

Also, do not expect that students will solve these problems in a single math segment. The emphasis is on the process, not the product. The goal is not only for a student to solve the problem, but rather, for them to be able to explain their thinking through pictures, words, and numbers. Whatever they put on paper should explain what they did well enough to allow someone else to understand it (Van de Walle & Lovin 2006).

Possible questions that engage students:

- Can students separate objects from a larger set of objects?
- Are students able to explain what happens when some objects are removed from a set of objects? Added to a set of objects?
- Can students explain how to find out what is left when one quantity is removed from another?
- Can students explain patterns as the story develops?

Have students write in their math journal about what was easy and what was difficult for them during today's problem solving. As you read (or listen to what they dictate), take note of *what* they write, not *how* they write it (Burns 2006).

FORMATIVE ASSESSMENT QUESTIONS

- What counting strategies did you use to separate (or combine) information?
- What is this story asking you to find out?
- What strategy will you use to solve this problem?
- Is there another way to solve this problem?

DIFFERENTIATION

Extension

- As students develop proficiency with solving each type of addition and subtraction structure, have them write their own problems for others to solve.

Intervention

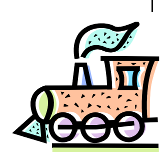
- Allow students to work with smaller numbers within 5 so that they can practice using efficient strategies to solve the problems. Counting strategies are efficient at this stage, but will become inefficient and distracting as numbers get larger. As students begin to understand the relationships among numbers, they will begin learning number facts at a recall level (Carpenter et al. 1999).

Additional Resources:

Carpenter, Thomas P., Fennema, Elizabeth, Loef Franke, Megan, Levi, Linda, Empson, Susan B., *Children's Mathematics: Cognitively Guided Math Instruction*: Heinemann, 1999.

PROBLEM TYPES

Join/Combine	Mouse had 6 cookies. Emma gave him 4 more. How many pennies does Mouse have? 6 + 4 = ?	Mouse had 6 pennies. Emma gave her some more. Now Mouse has 10 pennies. How many did Emma give her? 6 + ? = 10	Mouse had some pennies. Emma gave her 4 more. Now Mouse has 10 pennies. How many pennies did Mouse have to begin with? ? + 4 = 10
Separate/ Decompose	Mouse had 9 cookies. He gave 7 to Emma. How many cookies does Mouse have left? 9 – 7 = ?	Mouse had 9 cookies. He gave some to Emma. Now he has 2 cookies left. How many did he give to Emma? 9 - ? = 2	Mouse had some cookies. He gave 7 to Emma. Now he has 2 cookies left. How many cookies did Mouse have to start with? ? – 7 = 2
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[1]
Put Together / Take Apart[2]	Mouse has 3 chocolate chip cookies and 5 sugar cookies. How many cookies does he have? 3 + 5 = ?	Mrs. Haktak has 8 cookies. 3 are chocolate chip and the rest are sugar cookies. How many cookies does Mouse have? 8 – 3 = ?	Mouse has 5 cookies. How many are sugar and how many are chocolate chip? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	**("How many more?" version):** Spiders have 8 legs. Mice have 4 legs. How many more legs do spiders have than mice? **("How many fewer?" version):** Spiders have 8 legs. Mice have 4 legs. How many fewer legs do mice have than spiders?	**(Version with "more"):** Mice have 4 legs. Spiders have 4 more legs than mice. How many legs do spiders have? **(Version with "fewer"):** Mice have 4 fewer legs than spiders. Spiders have 8 legs. How many legs do mice have?	**(Version with "more"):** Spiders have 8 legs. They have 4 more legs than mice. How many legs do mice have? **(Version with "fewer"):** Mice have 4 fewer legs than spiders. Spiders have 8 legs. How many legs do mice have?



<u>CONSTRUCTING TASK</u>: **Building Trains** (Addition/Combining)

Approximately 1 day and repeated through centers (adapted from Chris Confer's "Snap It" found in Teaching Number Sense)

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

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MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
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8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"When parts of a set are known, addition is used to name the whole in terms of the parts." (Van de Walle & Lovin 2006) *Building Trains* allows children to focus on a single number for the entire activity. It is important to give the children time to work on a single number (usually to 4 or 5, at first) throughout the activity, allowing them opportunities to explore through a variety of materials and methods of joining or separating. As their understanding of concepts develops, encourage students to extend their understanding with higher numbers. Allowing students multiple opportunities to participate in these types of activities, gives them the chance to think about number relationships in a relaxed setting (Burns 2007; Van de Walle & Lovin 2006).

This activity reinforces the concept of addition (and the inverse, subtraction) through part-part-whole models. For students to see the two parts and the whole, the two parts must be kept as two separate parts. For example, if using counters instead of connecting cubes, the two groups should be kept in separate piles or separated by color and/or using a part-part-whole mat.

ESSENTIAL QUESTIONS

- What happens when I join quantities together?
- What happens when some objects are taken away from a set of objects?
- How can I find the total when I put two quantities together?
- How can I find what is left over when I take one quantity away from another?
- How can I compare one quantity to another?
- How can I solve problems using objects, pictures, words, and words?
- How can I use models to represent addition and subtraction?
- What happens when sets are joined or separated?

MATERIALS

- Connecting cubes (up to 10 per student) or other manipulatives (2-sided counters, shapes, dominoes, to name a few.)
- *Ten Black Dots* (Donald Crews 1986) or similar counting book

GROUPING

Whole, Individual, Small group task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part 1 (Addition)

Before reading the book, *Ten Black Dots* by Donald Crews, or a similar book involving addition (see literature list for suggested titles), ask students to turn to a partner and show them "10" using their "math hands" (notice students who can show ten immediately and those who may count each finger or count on from 5 fingers). During reading, as you come to each number, ask students how many more black dots you would need to get to ten. (For example, on the page that says "*…Two dots can make the eyes of a fox or the eyes of keys that open locks.*" Say, "I see two black dots, how many more dots are needed to make ten?")

Part II

After reading, begin an investigation to explore number relationships within 5 (and later within 10). Give each student five connecting cubes. Explain that when you say, "Break", that is their cue to break the train of cubes into two equal parts. When they have separated their train of cubes, they will need to notice how many are in each group and share the combination they made with the class. Model for them how to share the combination ("I had 5 cubes. As I break my train in half, I am left with a set of 3 and a set of 2. So, 2 and 3 is equal to 5."). Practice the activity with the class and have a few students share their combinations. Once you see that students understand the activity, assign partners and give each set of partners a sum of cubes to work with according to their abilities (numbers within 5 or 10). Students will take turns breaking their trains in two parts and show them to their partners. The partner will then say the combination aloud and repeat the process after switching roles.

Have students write in their math journal about the activity they participated in today. Have them write down any patterns they noticed during the activity ("3 + 2 is the same as 2 + 3. So, it doesn't matter the order of the addends, the sum is the same.") and allow them to share their thinking. For students that have difficulty writing their math thinking, allow them to dictate to you or someone who can scribe their thoughts.

Part III (Five Tower Game)
"Five-Tower Game" (Burns 2007) Students will roll 2 dice, add the numbers that come up and build a tower with that many connecting cubes. Each partner repeats the process until five towers are made. Once the towers are made, the two students compare the number of cubes each person has. How do they know who has more? Less? In addition, questions such as the following can increase engagement in what seems to be a low level activity.

What is the greatest amount of cubes you could earn?
What is the smallest amount of cubes you could earn?
What is the difference in two towers?
Can you use all your cubes and make 2 equal towers? Can you justify your thinking?

FORMATIVE ASSESSMENT QUESTIONS

- Does the order of the addends change the sum? Explain your thinking.
- How do you know when your answer makes sense?

DIFFERENTIATION

Extension

- "Two out of Three" Game (Activity 2.17, page 50. Van de Walle)

Intervention

- Allow students to work with numbers that are appropriate for their performance level. If students are still counting using one-to-one correspondence, you may suggest that they use numbers smaller than five. For those that seem to quickly identify number relationships to five, suggest numbers to ten until they can quickly identify number relationships within ten.
- Allow students who are ready to record addition sentences to describe how they broke apart their trains.

Building Trains (Addition) Name:____

	Is the same as		+	
	=		and	
	is equal to		+	
	and		Is the same as	
	+		=	
	+		=	

<u>CONSTRUCTING TASK</u>: **Building Trains** (Subtraction)

Approximately 1 day and repeated through centers (adapted from Chris Confer's "Snap It" found in <u>Teaching Number Sense</u>)

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"In part-part-whole model, when the whole and one of the parts are known, subtraction names the other part." (Van de Walle & Lovin 2006) Even when there is a remove action, the situation in this next activity ends with two clearly distinct parts. It is important for you to have a discussion with your students, which includes showing them how an addition and subtraction sentence can be written for the same situation, to help them connect addition and subtraction. (More information can be found in Chapter 3 of <u>Teaching Student-Centered Mathematics: Grades K-3</u>, Van de Walle & Lovin 2006)

ESSENTIAL QUESTIONS

- What happens when I join quantities together?
- What happens when some objects are taken away from a set of objects?
- How can I find the total when I put two quantities together?
- How can I find what is left over when I take one quantity away from another?
- How can I compare one quantity to another?
- How can I solve problems using objects, pictures, words, and words?

- How can I use models to represent addition and subtraction?
- What happens when sets are joined or separated?
- How can I use models to represent subtraction?

MATERIALS

- Connecting cubes (up to 10 per student) or other manipulatives (2-sided counters, shapes, dominoes, to name a few.)
- *Ten Black Dots* (Donald Crews 1986) or similar counting book

GROUPING

Whole, Individual, Small group task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Review the activity of *Breaking Trains* from the previous lesson. Ask them to share what patterns they may have noticed while working with the same number. After several students have had an opportunity to share, explain that this time, students will break their train and hide one of the parts behind their back. Their partner will have to identify the part shown and then predict the quantity of cubes that are behind their back. Once a prediction has been made, the partner with the cubes reveals how many are hidden for the other partner to check his or her thinking. They should both attempt to write the addition and subtraction sentence that describes the situation.

Before beginning, model the activity for the group. You will be “Player A” and the class will act as “Player B”. Using a set of ten, break the train and hide one part behind your back. All students will use their math hands to predict how many are behind your back (scan to see who has made a logical prediction and those whose predictions are illogical. Intervention may be needed for those that demonstrate a lack of number sense.). After all students make a prediction with their math hands, reveal the number of cubes behind your back. Next, ask a volunteer to show the class how to write an addition and subtraction number sentence that would describe the situation you modeled. Encourage them to share their math thinking. Repeat this process until you feel that students are ready to practice this same activity with a partner.

Once students are ready, assign each set of partners a number that is appropriate for their performance level and allow them to begin working. As you walk around and observe partners working together, stop and ask students some of the following questions (or similar questions to assess student learning)

- (as students show one part of cubes and hide the other part) What goes with (known part) to make _ (known whole) __? Or _ (known part) _ and ? make _ (known whole) _?
- What strategies did you use to find your answer?

Have students write in their math journal (or share aloud) the strategies they used to determine how many cubes were missing. Have them write down any patterns they noticed during the activity ("3 + 2 is the same as 2 + 3. So, it doesn't matter the order of the addends, the sum is the same.") and allow them to share their thinking. For students that have difficulty writing their math thinking, allow them to dictate to you or someone else who can scribe their thinking.

FORMATIVE ASSESSMENT QUESTIONS

- Does the order of the addends change the sum? Explain your thinking.
- How do you know when your answer makes sense?

DIFFERENTIATION

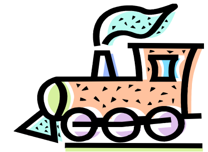
Extension

- "Grab Bag Subtraction"-adapted (Burns 2007) Children can work in pairs or independently for this activity. Fill a paper bag with any set of tiles or cubes (to 5 or 10) and write the quantity of items in the bag on the front of the sack. Students will reach in the bag and take some out, showing how many were removed. Both students will predict how many they think are left inside of the bag. Then they will check their predictions and record the addition and subtraction number sentences.

Intervention

- Students may use a part-part-whole mat for this activity and cover an addend with a sheet of paper. Make sure that students flip the mat so that the "whole" section is first, showing that the one part is removed from the whole. A part-part-whole mat can be as simple as the table below on white paper.

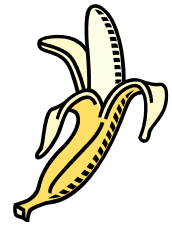
whole	
part	**part**



Building Trains (Subtraction)

Name:________________________________

	Is the same as		-	
	=		and	
	is equal to		-	
	minus		Is the same as	
	-		=	
	-		=	



PRACTICE TASK: Going Bananas

Approximately 1-2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students need to develop an understanding of part-whole relationships as they recognize that a set of objects (5) can be broken into smaller sub-sets (3 and 2) and still remain the total amount (5). In addition, this objective asks students to realize that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decompose), students use the understanding that a smaller set of objects exists within that larger set (inclusion).

Use of the recording sheet for this task is optional because students need to learn to draw a representation of what they are thinking for three reasons:

SMP2. Reason abstractly and quantitatively.
SMP3. Construct viable arguments and critique the reasoning of others.
SMP4. Model with mathematics.

By encouraging students to organize their work and draw representations, students can begin to "talk through" their process. This enables students the opportunity to attend to precision as they explain and reason mathematically. Using tables, numbers, pictures and words allows students to become more efficient when identifying patterns in numbers and making generalizations.

Commutative property might be identified during this task. Have students discuss whether or not to include commutative properties as possible combinations. There is no right or wrong answer, students should construct viable arguments and explain why their combination of bananas is correct. In addition students must be willing to analyze and critique the reasoning of others.

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?
- How can I use different combinations of numbers to represent the same quantity?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5?

MATERIALS

- Paper
- Pencil
- Accessible manipulatives
- 5 bowls per group/student

GROUPING

Whole, Individual, Small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

It is useful to think of problem-based lessons as consisting of three main parts: *before, during, and after*. If you allot time for each part it is quite easy to devote a full period to one seemingly simple problem. (Van de Walle p.15-19)

<u>The Before Phase:</u> get students mentally prepared for the task, be sure the task is understood, and be certain that you have clearly established expectations beyond simply getting an answer.

<u>The During Phase:</u> the most important thing here is *let go!* Give students a chance to work without your guidance. Give them an opportunity to use *their* ideas and not simply follow directions. Your second task is to listen. Find out how different children or groups are thinking,

what ideas they are using, and how they are approaching the problem. In this phase hints may be provided but not solutions, and students should be encouraged to test ideas.

The After Phase: this is often where some of the best learning takes place. During the after phase students share emerging ideas and the community of class learners is developed. This will not develop quickly or easily and will be developed over time.

(*For a more detailed description of the three-phase problem-based learning model see Van de Walle Teaching Student-Centered Mathematics K-3 pp 15-19)*

The task presented to students is as follows:

> *The monkeys loved to pick bananas. They loved to count their bananas and put them into baskets. If the monkeys picked 5 bananas one morning, what are all the possible ways they could put the bananas into baskets? Explain and justify.*

Possible solutions for decomposing 5 (The concept of commutative property can be explored but only if it is discovered by students.

$5 = 1 + 1 + 1 + 1 + 1$
$5 = 2 + 1 + 1 + 1 + 0$
$5 = 2 + 2 + 1 + 0 + 0$
$5 = 3 + 1 + 1+ 0 + 0$
$5 = 3 + 2+ 0 + 0 + 0$
$5 = 4 + 1+ 0 + 0 + 0$
$5 = 5 + 0 + 0 + 0 + 0$

Comment: Although the standard addresses pairs of numbers to 5, teaching open ended word problems in a context allow students to go above and beyond the expectation of the standards. As a possible closing or follow up to this task, ask students to identify only the pairs of numbers that make 5 when combined. This can be achieved by limiting the number of bowls to 2.

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more was to decompose the number 5? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!
- What is the most number of baskets the monkeys could use?
- What is the fewest number of baskets the monkeys could use?

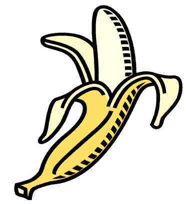
DIFFERENTIATION

Extension

- Students can be encouraged to find the solution for multiple quantities of bananas. In addition, students can create a story problem using a quantity of 5.

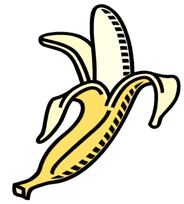
Intervention

- Allow students to work through the stages at a speed that is appropriate for their performance level. Some students may need additional experiences acting out problems, using manipulatives, or drawing pictures.
- Students can use cut out pictures of bananas and act out the story by placing the cut outs in bowls or pictures of bowls.
- Start with a smaller number than 5 and allow students to build and generalize with the patterns that they are seeing. In addition, you could limit the number of baskets that a student uses.



Going Bananas

The monkeys loved to pick bananas and put them into baskets. If the monkeys picked 5 bananas one morning, what are all the possible ways they could put their bananas into baskets? Explain and justify.



Going Bananas

The monkeys loved to pick bananas and put them into baskets. If the monkeys picked 10 bananas one morning, what are all the possible ways they could put their bananas into baskets? Explain and justify.

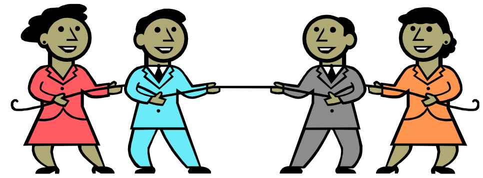










PRACTICE TASK: Tug-O-War

Approximately one day and repeated through centers

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Many children will use number lines (models) to solve story problems. The model is a thinking tool to help them both understand what is happening in the problem and a means of keeping track of the numbers and solving the problem. Tug-O-War is a task that helps students become familiar with adding and subtracting along number line. (Van de Walle p.72-72)

ESSENTIAL QUESTIONS

- How can using benchmark numbers help me when adding or subtracting?
- How can I use models to represent addition and subtraction?
- How can I find what is left over when I take one quantity away from another?

MATERIALS

- 1 counter
- 1 Six-sided dice
- Tug-O-War game board

GROUPING

Partners (2 players)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Place the counter at the number 10 on the number line.

Player 1 roles the dice and moves the counter the correspoding number of spaces towards zero on the number line. (If the chip is on ten and player 1 rolls a 3 they move the chip 3 spaces to 7) Player one must identify and say the number of where the chip is on the number line. If player 1 is unable to correctly identify the correct location of the chip, it moves back to the previous location.

While the game is being played students should use a double ten frame to model what is happening to the counter on the number line which in turn will help with strategy development. Although writing the equation is NOT a standard that is explicitly listed in Kindergarten, it is suggested that students be provided with lots of opportunities to record their thinking. Students can record the results of their turn in their journal.

Player 2 roles the dice and moves the counter that many spaces towards 20. (If the chip is on 7 and player 2 rolls a 6, they move the chip six places to 13) If player 2 is unable to identify the correct location of the chip, it moves back to the previous location.

If the chip reaches zero on the number line, player 1 wins. If the chip reaches 20 on the number line, player 2 wins.

Comment: As students are engaged in Tug-O-War, observe how students move the counter to locate the new place on the number line. Is the student counting by ones, or are they using a strategy? If so, which one? When students are locating the counter on the number line, do they need to start at 0 and count up, or are the able to use the benchmark numbers to count forward or backwards to determine the location?

FORMATIVE ASSESSMENT QUESTIONS

- Instead of counting 1 by 1, could you have located the counter a different way?
- Are you using any strategies to help you find where the counter should be placed?
- Did you use the benchmark numbers to locate the place on the number line?
- Where is the counter located on the number line right now? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns when playing the game? Explain!

DIFFERENTIATION

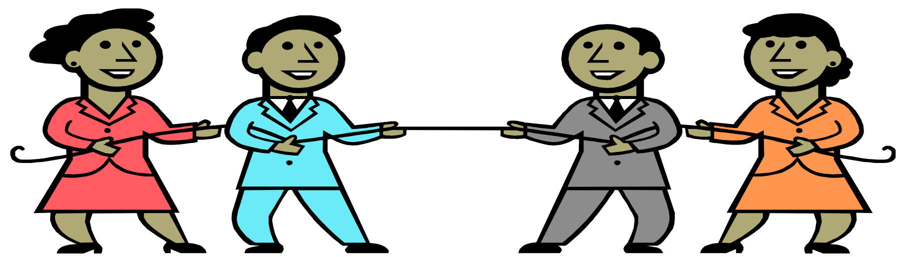
Extension

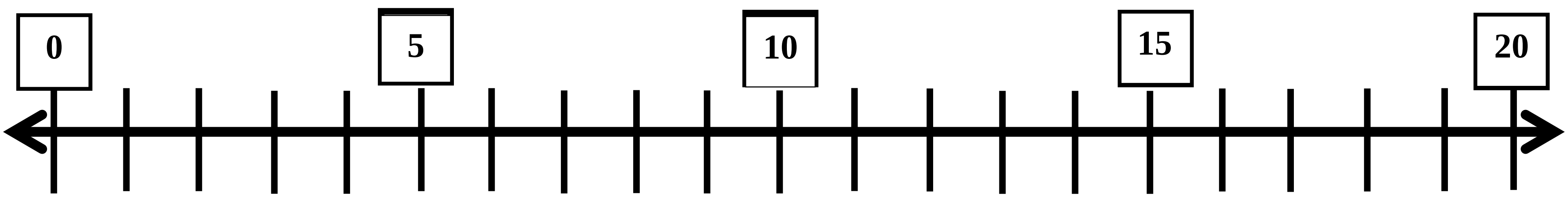
- 2 sixed-sided diced could be used to extend the game. Have players roll the dice, determine the difference and move the corresponding number of spaces

Intervention

- The numerals can be written on the *Tug-O-War* game board to help assist students that struggle with identifying the location on the number line.

Tug-O-War



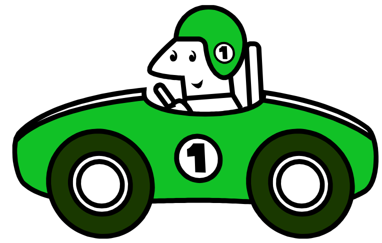


Materials

1 counter and six-sided dice (1-6) or spinner numbered (1-3)

Directions

1. Place the counter at the number 10 on the number line.
2. Player 1 roles the dice and moves the counter the correspoding number of spaces towards zero on the number line. (If the chip is on ten and player 1 rolls a 3 they move the chip 3 spaces to 7) Player one must identify and say the number of where the chip is on the number line. If player 1 is unable to correctly identify the correct location of the chip, it moves back to the previous location.
3. Player 2 roles the dice and moves the counter that many spaces towards 20. (If the chip is on 7 and player 2 rolls a 6 they move the chip six places to 13) If player 2 is unable to correctly identify the correct location of the chip, it moves back to the previous location.
4. If the chip reaches zero on the number line, player 1 wins. If the chip reaches 20 on the number line, player 2 wins.



PRACTICE TASK: Race To 20 Revisited

Approximately one day and repeated through centers

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In this task students will roll the dice and make a 5. If a student rolls a 3 they would need 2 more to make a 5, so they move 2 spaces. Students will eventually roll a 6. This will provide an opportunity for class discussion and for students to engage in the SMPs. In order to make a 5 from 6, one would need to be removed. Have students discuss how this could be represented in the game (move backwards one space).

ESSENTIAL QUESTIONS

- How can using benchmark numbers help me when adding or subtracting?
- Why is it important that I can build the number combinations for the number 5?
-

MATERIALS

- 2 different colored counters
- 1 number cube (1-6)
- Race to 20 game board

GROUPING

Partners (2 players)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Each player places their counter in the starting square.

Player one rolls the dice and moves as many spaces needed to make a 5. Example: 3 rolled, move forward 2 places. 4 rolled move forward 1.

Players must correctly identify what number square they are on and how they know or return back to the previous square.

A player alternate turns until one player reaches 20.

As students are engaged in Race to 20, observe how students move the counter to locate the new place on the game board. Is the student counting by ones, or are they using a strategy? If so, which one? When students are locating the counter on the number line, do they need to start at 0 and count up, or are the able to use the benchmark numbers to count forward or backwards to determine the location.

FORMATIVE ASSESSMENT QUESTIONS

- Instead of counting 1 by 1, could you have located the counter a different way?
- Are you using any strategies to help you find where the counter should be placed?
- Did you use the benchmark numbers to locate the place on the board?
- Where is the counter located on the board right now? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns when playing the game? Explain!

DIFFERENTIATION

Extension

- **(2)** sixed-sided diced could be used to extend the game. Have players roll the dice, determine the difference and move the corresponding number of spaces

Intervention

- While the game is being played students can use a ten-frame to model what is happening to the counter. This can also be beneficial for students that struggle to identify strategies.

Start					5 Great Tire Change… move another 5				

10

Materials:
2 different colored counters
1 number cube (1-6)

Directions:
-Each player places their counter in the starting square.
-Player one rolls the dice and moves as many spaces needed to make a 5. Example: 3 rolled move forward 2 places. 4 rolled move forward 1.
-players must correctly identify what number square they are on and how they know or return back to the previous square.
-Players alternate turns until one player reaches 20.

Race to 20

Finish  20					15	Flat Tire miss a turn			

<u>CONSTRUCTING TASK:</u> Shake and Spill Addition (5 Or 10)

Approximately one day and repeated through centers (adapted from Marilyn Burns' Shake and Spill activity found in <u>About Teaching Mathematics: A K-8 Resource</u>

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.3.Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4.For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5. Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"When parts of a set are known, addition is used to name the whole in terms of the parts." (Van de Walle & Lovin 2006) *Shake and Spill* allows children to focus on a single number for the entire activity. It is important to give the children time to work on a single number (usually to 4 or 5, at first) throughout the activity, allowing them opportunities to explore through a variety of materials and methods of joining or separating. As their understanding of concepts develop, encourage students to extend their understanding with higher numbers. Allowing students multiple opportunities to participate in these types of activities gives them the chance to think about number relationships in a relaxed setting (Burns 2007; Van de Walle & Lovin 2006).

This activity reinforces the concept of addition (and the inverse, subtraction) through part-part-whole models. For students to see the two parts and the whole, the two parts must be kept as two separate parts.

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- Two-color counters
- One small cup, per student (like a mouthwash cup)
- One dry-erase marker per student
- Shake and Spill recording sheet for addition
- *Anno's Counting House* by Mitsumasa Anno, or another similar counting book

GROUPING

whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part 1
Begin this task by "reading" the math picture book, *Anno's Counting House* (Anno 1982) or a similar book. As some of the children in the story move to the other house, ask questions of the group such as, "*Who's gone? How many are gone? If ____ are gone, how many must be in the first house*?" With rich conversation during this story, all combinations of ten will be discussed.

After reading, begin an investigation to explore number relationships within 5 (and later within 10). Give each student a cup of five double-sided counters. Ask students to swirl the cup of counters and then spill them on their table or work space. How many red counters do you see? How many yellow? Then show students how to record what they see using a number sentence. Allow them to write on their table, using their dry-erase marker, the same number sentence. Repeat this process until you see that a majority of your students are ready to practice on their own.

When showing students how to record a number sentence, make sure to identify all the parts: the addends, the equal sign (can be read, "the same as"), and the answer to an addition number sentence, or sum. Take care to ensure that students can differentiate between the term "sum" and the word "some". Once students demonstrate an understanding of the activity, allow them to do the activity on their own. Assign each student a number of counters to work with according to his or her ability level (5 or 10). Each student will swirl and spill the counters in his cup, draw what they see, and write a number sentence that describes what is shown.

Possible solutions: 5= 0+5, 0+5=5, 4+1=5, 1+4=5, 3+2+5, 2+3=5
10+0=10, 0+10=10, 9+1=10, 1+9=10, 8+2=10, 2+8=10, 7+3=10, 3+7=10, 6+4=10, 4+6=10, 5+5=10

Possible questions that engage students:
When recording the number of red sides and yellow sides each time you spill the counters, do you think you'll get one result more often than others?
If so, what will it be? How do you know?

Have students write in their math journal about the activity they participated in today. Ask them to list how many number combinations they were able to make for 5 or 10. What did they notice about those combinations?

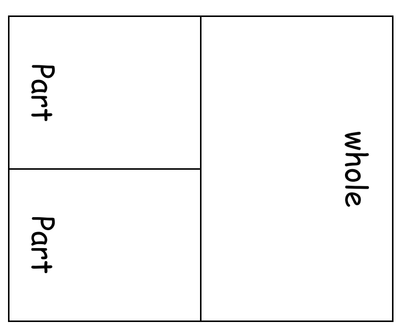
FORMATIVE ASSESSMENT QUESTIONS

- Does the order of the addends change the sum? Explain your thinking.
- How do you know when your answer makes sense?

DIFFERENTIATION

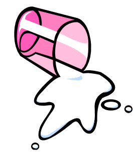
Extension
- Play, "Five-Frame Tell-About" (Van de Walle, page 46). This game could also be changed to "Ten-Frame Tell-About".

Intervention
- Allow students who have difficulty with organization to use a 5 or 10s frame. As they spill the counters, have them fill up the frames with the counters and then record their findings.
- Students may use a part-part-whole mat for this activity. A part-part-whole mat can be as simple as the table below on white paper.



Shake and Spill: Addition

Number of Red Counters	+	Number of Yellow Counters	= is the same as:	Total Number of Counters



<u>CONSTRUCTING TASK:</u> Shake and Spill Subtraction (5 Or 10)

Approximately one day and repeated through centers (adapted from Marilyn Burns' Shake and Spill activity found in <u>About Teaching Mathematics: A K-8 Resource</u>

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.3.Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4.For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5. Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"In part-part-whole model, when the whole and one of the parts are known, subtraction names the other part." (Van de Walle & Lovin 2006) Even when there is a remove action, the situation in this next activity ends with two parts clearly distinct. It is important for you to have a discussion with your students, which includes showing them how an addition and subtraction sentence can be written for the same situation, to help them connect addition and subtraction. (More information can be found in Chapter 3 of <u>Teaching Student-Centered Mathematics: Grades K-3</u>, Van de Walle & Lovin 2006)

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.

- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- Two-color counters
- One small cup, per student (like a mouthwash cup)
- One dry-erase marker per student
- Shake and Spill recording sheet for subtraction
- *Anno's Counting House* by Mitsumasa Anno, or another similar counting book

GROUPING

whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Review the activity of *Shake and Spill* from the previous lesson. Ask students to share what patterns they may have noticed while working with the same number. After several students have had an opportunity to share, explain that this time, students will write the total number of counters they begin with and then subtract the number of yellow counters that are spilled.

Before beginning, model the activity for the group. You will be "Player A" and the class will act as "Player B". Using a set of five or ten, spill the counters in the cup and identify how many yellow counters are shown. Then think aloud, "*So, if I started off with 5 (or ten) counters, and there are yellow spilled, how many red counters were spilled*? *How many more do I need to get to 5 (or 10)?*" All students will use their math hands to predict how many red counters were spilled (scan to see who has made a logical prediction and those who's prediction are illogical. Intervention may be needed for those that demonstrate a lack of number sense.). After all students make a prediction with their math hands, reveal the number of red counters that were spilled. Next, ask a volunteer to show the class how to write an addition and subtraction number sentence that would describe the situation you modeled. Encourage them to share their math thinking. Repeat this process until you feel that students are ready to practice this same activity independently (or with a partner).

Once students are ready, assign each student a number that is appropriate for their performance level and allow them to begin working. As you walk around to assess students, stop and ask the following questions (or those similar to assess student learning):

- What number did you start off with? So, if there are ___ yellow counters, how many red counters do you think there are? How do you know without counting?

Have students write in their math journal (or share aloud) the strategies they used to determine how many red counters were spilled. What do they notice about this version of the game that is alike or different from the previous version?

FORMATIVE ASSESSMENT QUESTIONS

- Does the order of the addends change the sum? Explain your thinking.
- How do you know when your answer makes sense?

DIFFERENTIATION

Extension

- "Grab Bag Subtraction"-adapted (Burns 2007) Children can work in pairs or independently for this activity. Fill a paper bag with any set of tiles or cubes (to 5 or 10) and write the quantity of items in the bag on the front of the sack. Students will reach in the bag and take some out, showing how many were removed. Both students will predict how many they think are left inside of the bag. Then they will check their predictions and record the addition and subtraction number sentences.

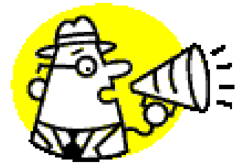
Intervention

- Students may use a part-part-whole mat for this activity. Make sure that students flip the mat so that the "whole" section is first, showing that the one part is removed from the whole. A part-part-whole mat can be as simple as the table below on white paper.

<table>
<tr><td rowspan="2">whole</td><td>part</td></tr>
<tr><td>part</td></tr>
</table>

Shake and Spill: Subtraction

Number of Red Counters	-	Number of Yellow Counters	= is the same as:	Total Number of Counters



PRACTICE TASK: SUM-moning 5's and 10's

Approximately one day and repeated through centers

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.3.Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5.Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students can develop and refine strategies as they hear other students' descriptions of their thinking about number combinations. For example, a student might compute 8 + 7 by counting on from 8: "..., 9, 10, 11, 12, 13, 14, 15." But during a class discussion of solutions for this problem, she might hear another student's strategy, in which he uses knowledge about 10; namely, 8 and 2 make 10, and 5 more is 15. She may then be able to adapt and apply this strategy later when she computes 28 + 7 by saying, "28 and 2 make 30, and 5 more is 35" (NCTM Principles and Standards, 2012).

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?

- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- 1 SUM-moning FIVES game board, per pair of students
- 5 double-side counters, per student
- 2 dice, per pair of students
- five frame

GROUPING

Partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part 1
The first player rolls two dice and finds the sum.

After finding the sum of the two numbers, the student will decide what can be added to 5 to make the same sum. After a prediction has been made and explained, students model their explanation on the five frame board. If the student was correct, they place a double- sided counter over the equation that matches their prediction, and five frame explanation.

If the equation is not written, or if 5 cannot be used in an equation that is on the board, the player skips a turn to fill the board.

They two players alternate turns until the first player fills up his game board.

The person with the board filled first, wins!

Part II
Have students share with the whole group the strategies they used to identify how many more were needed to make a number (5 or 10). Were there situations in which they added more than two numbers or that they subtracted two numbers? If so, what were they? Challenge students to identify all the roll combinations where 5 or 10 can’t be made with 2 dice.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you going to use to find the sum of two numbers?
- If you found sums higher than 5, what strategies did you use?

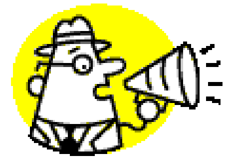
DIFFERENTIATION

Extension
- Change the game board to SUM-moning TENS and follow the same procedures.

- Activity 2.20, page 51 (Van de Walle & Lovin, 2006), "I Wish I Had". After modeling this activity for students, have them work in pairs to play.

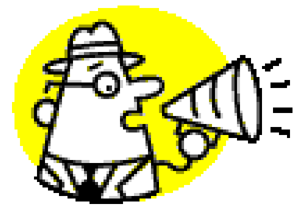
Intervention

- This game board can be changed to include only number combinations that are less than 5 or 10, for students who are not yet ready to work with sums beyond 5 or that are not yet proficient with using 5 as a benchmark.



SUM-moning 5's

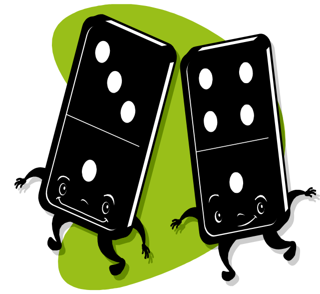
5	5
5+1	5+1
5+2	5+2
5+3	5+3
5+4	5+4



SUM-moning 10's

10
10+1
10+2
10+3
10+4
10+5

10
10+1
10+2
10+3
10+4
10+5



PRACTICE TASK: Bo Beep's Domino Dilemma

Approximately 1-2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Numbers are related to each other through a variety of number relationships. The number 7, for example, is 3 more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots. These ideas further extend to an understanding of 17, 99, and beyond. Number concepts are intimately tied to the world around us. Application of number relationships to the real world marks the beginning of making sense of the world in a mathematical manner (Van de Walle, 2010).

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?

- How can I use different combinations of numbers to represent the same quantity?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5?

MATERIALS

- Recording sheet (optional)
- Accessible manipulatives or dominos

GROUPING

whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
If students have never been introduced to the game of dominoes, allow students to explore and play a couple of games before the task begins. Have students discuss the strategies they used during the game.

Part II Memory
All the dominoes are placed face down. In partners, students take turns flipping over two dominoes where the total number of pips on the dominoes match. Students must find two different dominoes that have the same total (Example: one domino could have 3&2 and the matching domino could have 4&1). Once all dominoes have been matched up and no more matches are possible, the player with the most pairs of dominoes wins.

Part III
Match players by ability level and have them take turns flipping over a domino. The first player to call out the total sum of pips on the domino wins. However, in order for the player to collect the domino, they must describe the strategy they used to their partner. (Example: 5&2 Student says "7, and I know that because 2 more than 5 is 7. If the domino was 3&0 the student might say "3, and I know that because any number plus zero is the number")

After all the dominoes have been played, the player with the most dominoes wins.

Part IV (to 5)
Introduce students to the task of 5 pips. Begin by discussing the vocabulary in the task and check for understanding.

> *Little Bo Peep loved to play dominoes. She had one domino in her pocket. The sum of the pips on her domino was 5. What could the domino in Bo Peep's Pocket look like?*

Comment: there are more than enough squares provided for the task to 5. This is used to make students justify that they have found all of the possible combinations. Many times students will know they are complete when all the squares have been "filled up", so by providing more than enough squares, this cannot occur.

Part V (to 10)
Introduce students to the task of 10 pips. Begin by discussing the vocabulary in the task and check for understanding.

Little Bo Peep loved to play dominoes. One day she had 2 dominoes in her pocket and the sum of the pips on her two dominos was 10. What could have been the two dominoes in Bo Peep's Pocket?

Comment: there are not enough squares provided for this task. Any additional combinations to ten that students identify should be recorded in their math journal.

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more was to decompose the number 5? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension
- Have the students identify which number of pips is the most frequent amongst the dominos. Have students make a make a conjecture (draw a conclusion) explaining why the number is most frequent.

Intervention
- Use a blank domino mat or folded piece of paper and give the student 5 or 10 counters. Have students create the domino using the template and counters. Students then transfer model to the recording sheet.

Bo Peep's Domino Dilemma

Little Bo Peep loved to play dominoes. One day she had a domino in her pocket and the sum of the pips on her domino was 5. What could the domino in Bo Peep's pocket look like?

Domino	Total
______ ______	
______ ______	
______ ______	

Domino	Total

Domino	Total

Bo Peep's Domino Dilemma

Little Bo Peep loved to play dominoes. One day, she had 2 dominoes in her pocket and the sum of the pips on her two dominoes was 10. What could have been the two dominoes in Bo Peep's Pocket?

Domino #1	Domino #2	Total
______ ______	______ ______	
______ ______	______ ______	
______ ______	______ ______	

Domino #1	Domino #2	Total
______ ______	______ ______	
______ ______	______ ______	
______ ______	______ ______	
______ ______	______ ______	

PERFORMANCE TASK: The Bike Shop

Approximately 1-2 Days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In developing the meaning of operations, teachers should ensure that students repeatedly encounter situations in which the same numbers appear in different contexts. For example, the numbers 3, 2, and 5 may appear in problem-solving situations that could be represented by 2 + 3, or 3 + 2, or 5 – 3, or 5 – 2. Although different students may initially use quite different ways of thinking to solve problems, teachers should help students recognize that solving one kind of problem is related to solving another kind. Recognizing the inverse relationship between addition and subtraction can allow students to be flexible in using strategies to solve problems (NCTM Principles and Standards, 2012).

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?
- How can I use different combinations of numbers to represent the same quantity?
- How can I represent problems using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5?

MATERIALS

- The Bike Shop Recording sheet
- Bicycle cut-outs (if necessary)

GROUPING

Whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

Begin by discuss bikes and bike shops with students. Allow students to share any experiences they have had with bikes. Specifically, discuss different types of bikes and what makes them different (focus on the wheels in particular). Ask the students the question "if I have 3 wheels sitting at home in my garage, what types of bikes could be in my garage?" Students will immediately say a tricycle but have them mentally explore possible combinations of bikes (unicycle and bike, for example).

Part II

Present students with the following task being sure to go over terms and understanding. Many students will know what a display case is, but not the formal name for it.

> *The Bike Store sold unicycles, bicycles, tricycles and go-carts. The owner of the store could only fit a total of 5 wheels in the display case at the front of the store. What are the different combinations of bikes and go-carts that could be put on display in the front of the store? Explain and justify your mathematical thinking.*

Part III

Begin this part by allowing students to explore the possible combinations of bike wheels for number less than 10. Ask the students "if there were 5 wheels and 3 seats in the front display case, what could be in the display?" Students will struggle more with this concept because of the addition of a specific number of seats. As students work through this task watch for students that have more or less than 3 types of bikes. This would indicate that students don't understand the purpose of the seats and how the number must remain constant. After students have solved this problem present the following task:

The Bike Store sold unicycles, bicycles, tricycles and go-carts. In the front display case they are 10 wheels and 6 seats. What types of bikes are on display in the front of the Bike Shop? Explain and justify your mathematical thinking.

FORMATIVE ASSESSMENT QUESTIONS:

- Are there any more was to decompose the number 5 or 10? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a strategy to find your answers?
- Did you identify any patterns or rules? Explain!
- Are there any paired combinations to 5/10 that you were unable to model? If so, share your thinking.

DIFFERENTIATION

Extension

- Have students create a combination of bike wheels that are equal to 10. Partners play and try to identify the combination used by their partner.

Intervention

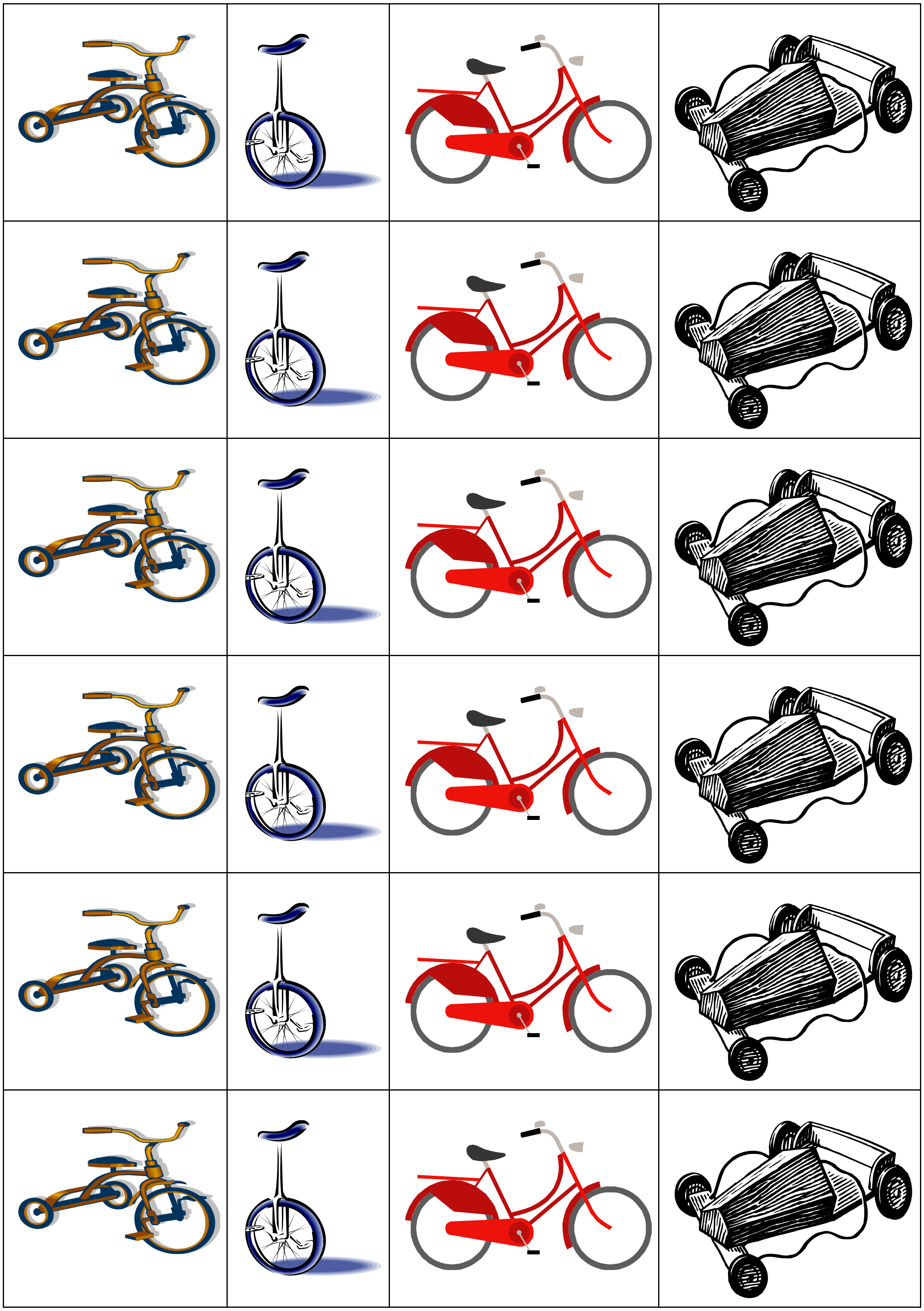
- Use clip art of unicycles, bicycles, tricycles, and go carts for students to manipulate and exchange one type of bike for another.
- For Part III (specific combination to 10), have students find any combinations to 10 that are possible and remove the restriction of 6 seats.

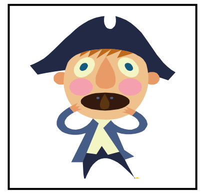
The Bike Shop

The Bike Store sold unicycles, bicycles, tricycles and go-carts. The owner of the store could only fit a total of 5 wheels in the display case at the front of the store. What are the different combinations of bikes and go-carts that could be put on display in the front of the store? Explain and justify your mathematical thinking.

The Bike Shop

The Bike Store sold unicycles, bicycles, tricycles and go-carts. In the front display case they are 10 wheels and 6 seats. What types of bikes are on display in the front of the Bike Shop? Explain and justify your mathematical thinking.





CONSTRUCTING TASK: Calling Out Reds!

Approximately one day and repeated through centers (adapted from the Marilyn Burns activity, How Many Reds?)

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"Children need many opportunities to identify quantities, see relationships between numbers, and learn about the operations of addition and subtraction. When developing beginning concepts of number, children benefit from exploring concrete materials and relating numbers to problem situations. They also benefit from talking about their ideas and hearing how other children think. The learning activities should be varied in their contexts and involve the children directly in thinking, reasoning, and solving problems." (Burns 2007)

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- *Calling Out Reds?* Recording sheets
- Two different colored dice, per set of partners
- *Anno's Counting House* by Anno Mitsumasa

GROUPING

Whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Refer to the book, *Anno's Counting House* (Anno 1982) read in a previous lesson or "re-read" the book but pose different questions, such as, "*What if the children moved in pairs instead of one at a time? What if there were three houses? What if there were more children? What else could be in the house to count? How many rooms, pictures, windows? What about your house? What about two classrooms or two buses instead of houses?" (Van de Walle & Lovin 2007)* Have the children use counters to work out the story according to the new questions you pose.

Select two students to model this game for the class (or select two pairs of students and divide your class in half, so that a more intimate setting is provided for students to observe each pair of students playing the game. You could facilitate one group, your paraprofessional and/or collaborative teacher could facilitate the other.). The children will need twenty playing cards in all, consisting of ace through 10 of a red suit and black suit along with a recording sheet.

One child will mix up the cards, and the deal them out, facedown, ten to a person. Each student will count the number of reds they have and write it on their recording sheet. Then, they will predict how many reds their partners has so that they make can make ten (show the students how to write the addition sentence). After each partner has written down the number of reds they have and the number of reds their partners has to equal ten, they will check their work by each revealing the number of red cards they have. Have the pairs repeat this process until you determine that the whole group is ready to play.

Once students are ready to play, have students work with their math partner for this activity. As you observe students, ask the following questions to probe their math thinking:

- Do you notice a pattern?
- What pair of addends do you seem to have the most of? The least of?

- How are you able to determine how many more you need to make ten, without counting? Or are you able to?

Invite each pair of partners to come to the whole group with their recording sheets. Make a class graph of the addends the children recorded. Analyze the graph and have students speculate about why some pairs of addends come up more often that others (Burns 2007).

Teacher Friendly Tip: Copy the addition and subtraction recording form back to back, laminate them (or use a sheet protector), and you have recording sheets that will last you a lifetime without making multiple copies! You will only need one class set. Students will use overhead markers to record their answers. If you have a student on whom you are collecting data, make a copy before they erase their work and you have documentation of progress.

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more ways to set the bikes in the display case? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension

- *Make 10 on the Ten-Frame* (Van de Walle, Activity 4.8; Van de Walle & Lovin 2006) This activity can be adapted by changing it to “Make 5 on the Five-Frame” to keep the sum within ten. Use fact cards that have sums less than ten and give each student two five-frames to use to model each number on the fact card. Then, students should decide on the easiest way to show what the total is, without counting. Take note as students explain their thinking of which strategies they are using-are they efficient strategies?

Intervention

- As students count their red cards, they could fill up a ten-frame with the corresponding number of counters. They will then record that number on their recording sheet and then identify how many more are needed to make ten. The student should then check their predictions by actually counting out the number of red cards their partner has.

Calling Out Reds?

Name:____________________________

Number of Red Cards I have	Number of Red Cards I *predict* my partner has	How many actual red cards my partner has	Total Number of Red cards

PRACTICE TASK: Dropping Pennies

Approximately 1-2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Numbers are related to each other through a variety of number relationships. The number 7, for example, is 3 more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots. These ideas further extend to an understanding of 17, 99, and beyond. Number concepts are intimately tied to the world around us. Application of number relationships to the real world marks the beginning of making sense of the world in a mathematical manner (Van de Walle, 2010).

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?

- How can I use different combinations of numbers to represent the same quantity?
- How can I represent problems using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5?
- How do you know when your answer makes sense?

MATERIALS

- Pencil
- Recording sheet and pennies

GROUPING

Whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Review or introduce a penny and discuss how both sides of the penny are different. Be sure that students understand which side is heads on the coin and which side is tails. Draw a chart that is similar to the *Dropping Pennies* recording sheet. Give each student 3 coins and have them explore all the ways that the coins can drop and record the possible combinations. DO NOT complete the chart. Take only a few combinations and then present students with Part II.

Part II
Comment: there are more squares provided on the recording sheet than actual combinations. This is so that students can justify their answer.

> *Sam dropped 5 pennies on the ground. Some were heads up and some were tails up. How might the pennies have fallen? Show all the ways the coins could have landed on the ground.*

In closing have students share the combinations they found and any strategies they used to solve the problem.

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more was to decompose the number 5? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a strategy to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension

- Instead of giving the student 5 pennies, they could use 5 coins that have total value of 20 cents or less. At the end of the task, students would be asked to add up their coins and justify their total value using numerals, pictures, and words. This could be recorded on the back side of the page or in the student's math journal.

Intervention

- Have the students act out the problem by dropping pennies on the ground and recording the result. No result can be repeated. Because all combinations may never be the result of acting out this task, have the student determine the missing solution(s) through questioning.

Dropping Pennies

Sam dropped 5 pennies on the ground. Some were heads up and some were tails up. How might the pennies have fallen? Show all the ways the coins have landed on the ground.

Heads	Tails	Total

Dropping Pennies

Sam dropped 10 pennies on the ground. Some were heads up and some were tails up. How might the pennies have fallen? Show all the ways the coins have landed on the ground.

Heads	Tails	Total

<u>PRACTICE TASK:</u> Make 5 (Or 10) Go Fish!

Approximately one day and repeated through centers

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"A mental computation strategy is simply an invented strategy that is done mentally. What may be a mental strategy for one student may require written support by another…As your students become more adept, they can and should be challenged from time to time to do appropriate computations mentally. Do not expect the same skills of all students." (Page 161: Van de Walle & Loving, 2006)

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?

- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- 1 deck of playing cards for each pair of students, face cards removed (ten frame cards will also work for this game)

GROUPING

partner or small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Engage students in a discussion about the possible number combinations to make 10. After students have explored possible combinations, introduce them to *Make 10 Go Fish.*

How to play: The object is to get two cards that total 10.

- Each player is dealt five cards. The rest of the cards are placed down in the center of the table.

- If you have any pairs of cards that total 10, put them down in front of you and replace those cards with cards from the deck.

- Take turns. On your turn, ask the other player for a card that will go with a card in your hand to make 10.

- If you get a card that makes 10, put the pair of cards down. Take another card from the deck. Your turn is over.

- If you do not get a card that makes 10, take the top card from the deck. Your turn is over. (Example: Player 1 *"Do you have a 2 in your hand?"* If player 2 has a 2 they give it to player 1. If they do not have a 2 they say "*Go Fish!")*

- If the card you take from the deck makes 10 with a card in your hand, put the pair down and take another card. Your turn is over.

- If there are no cards left in your hand but still cards in the deck, you take two cards.

- The game is over when there are no more cards.

- At the end of the game make a list of the number pairs you made.

Part II
Ask students for all the possible combinations to make 5. At first students will probably use addition as a means to make a 5. Guide the students to think of making a 5 through separating/decomposing larger numbers.

Using the same rules as *Go Fish 10*, have students play *Go Fish 5.* If a student had a 7 and they asked their partner for a 2 and received it, their number sentence could be "*I had a 7 and I decomposed the 7 into a 2&5*" OR "*7 minus 2 is equal to 5*"

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you going to use?
- What number are you using? What do you need to find out before you can ask your partner for a card?
- What if you could only use three cards to make 5 (or 10)? Which ones would you use?

- Have students share with the whole group the strategies they used to identify how many more were needed to make a number (5 or 10). Were there situations in which they added more than two numbers or that they subtracted two numbers? If so, what were they?

DIFFERENTIATION

Extension
- Encourage students who have a strong knowledge of number combinations to 5 and 10 to use number combinations with more than two numbers.

Intervention
- Allow students who are still at the direct modeling stage to use 5 or 10 frame cards, manipulatives, and/or write number sentences to show their thinking when trying to determine how many more are needed to make 5 or 10.

<u>Practice Task:</u> OLD FARMER MCDONALD

Approximately 1-2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, me drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In developing the meaning of addition and subtraction with whole numbers, students should also encounter the properties of operations, such as the commutativity and associativity of addition. Although some students discover and use properties of operations naturally, teachers can bring these properties to the forefront through class discussions (NCTM Principles and Standards, 2012).

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?
- How can I use different combinations of numbers to represent the same quantity?

- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- Recording Sheet
- Accessible manipulatives
- Pictures of barn animals or toy animals (not included)

GROUPING

Whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Engage students in a class discussion about the farm and possible animals that could be seen if they visited the farm. When discussing the animals, have students pay particular attention to the number of legs on each animal. Tell the students, "*I was driving on the weekend and saw 6 feet, but I couldn't see the animals' upper body because they were behind the tractor. If saw 6 feet, what animals could I have seen?"* Give students think time and allow them to share their combinations of animals with the person they were sitting next to. As students share their combinations, the teacher records them on the board or chart paper so that the rest of the class can justify that the combination is accurate.

Introduce students to the following task and check for student understanding of what is being asked of them.

> *Old Farmer McDonald put all of his animals into different barns each night. Each barn could have no more than 10 feet. What animals did Old McDonald put in his barn to equal ten feet? Find the different combinations of farm animals that equal 10 feet. Be sure to explain and justify your combinations.*

Part II (What's On My Farm)
Have students create a specific combination of animals on the farm and play *What's On My Farm*. Player 1 arranges a combination of animals and tells their partner how many legs and eyes they have on their farm in addition to the types of animals used.

- Example: The combination of animals on my farm is one goat and a chicken so I would tell my partner "I have 6 legs and 4 eyes on my farm. What do I have on my farm?"

- Example: The combination of animals on my farm is three goats and two chickens so I would tell my partner "I have 16 legs and 10 eyes on my farm. What do I have on my farm?" As students work through the extension ask them if they notice any patterns.

Comment: Students will know how many animals there are on the farm but looking at the number of eyes. (Example: if there were 6 eyes, there are 3 animals) This strategy should be identified by the students and not teacher prompted.

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more ways to decompose the number 5? How do you know?
- Why did you decide to do it this way?
- Are you sure that you have found all of the ways? Why do you think so? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension

- Give students 3 numbers (1, 5, and 10). Ask students to make the number 20 using only the given 3 numbers. Some possible combinations could be 10+10 or 10+5+1+1+1+1+1.

Intervention

- Print out pictures of farm animals or use animal figurines to manipulate while trying find out the correct combination of farm animals. You could also give students a specific number of animals that were in the barn which would limit the possibility of combinations.

Old Farmer McDonald

Old Farmer McDonald needed to put all of his animals into different barns each night. Each barn could have no more than 10 feet. What animals did Old McDonald put in his barn to equal ten feet?

Find the different combinations of farm animals that equal 10 feet. Be sure to explain and justify your combinations.

PRACTICE TASK: Four Sums in a Row

4

Approximately one day and repeated through centers

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

As your students play this game, encourage them to use what they know from previous activities about number combinations to 5 and 10, and also what they know about using 5 and 10 as benchmark numbers. Young children often initially compute by using objects and counting; however, prekindergarten through grade 2 teachers need to encourage them to shift, over time, to solving many computation problems mentally or with paper and pencil to record their thinking. Students should develop strategies for knowing basic number combinations (the single-digit addition pairs and their counterparts for subtraction) that build on their thinking about, and understanding of, numbers. Fluency with basic addition and subtraction number combinations is a goal for the pre-K–2 years (NCTM Principles and Standards, 2012).

ESSENTIAL QUESTIONS

- Does the order of addends change the sum? Give examples to justify your thinking.
- How do you know when your answer makes sense?
- How can I represent problem situations using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5? 10?

MATERIALS

- 1 Four Sums in a Row game board, per pair of students
- 2 clothespins
- 24 double-sided counters, per set of partners

GROUPING

Partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

The first player places the 2 clothespins at the bottom of the game board and covers two addends. Player combines the addends and places a yellow marker over the sum.

The other player moves **only one** of the clothespins to a new addend and covers the sum. *Both addends can cover the same addend*.

Play continues with pairs alternating turns, moving one neutral counter each turn until one player has four markers in a row (horizontally, vertically, or diagonally)

Have students share with the whole group the strategies they used to identify how many more were needed to make a number (5 or 10). Were there situations in which they added more than two numbers or that they subtracted two numbers? If so, what were they?

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you going to use?
- What strategies did you use to get four sums in a row?
- If you found sums 11 and/or 12, what strategies did you use?

DIFFERENTIATION

Extension

- Activity 2.20, page 51 (Van de Walle & Lovin, 2006), "I Wish I Had". After modeling this activity for students, have them work in pairs to play.

Intervention

- This game board can be changed to include only number combinations to 5 or 10 for students who are not yet ready to work with sums beyond 10 or that are not yet proficient with using 5 as a benchmark.

4

Four Sums in a Row

3	10	12	9	6
9	2	5	3	7
3	8	7	0	5
7	4	8	6	9
5	12	1	2	4

0 1 2 3 4 5 6 7 8 9

Four Sums in a Row

I kept the number...	I used the number...	Total

I kept the number...	I made the number...	Total

<u>PERFORMANCE TASK:</u> The Candy Store

Approximately 1-2 Days

STANDARDS FOR MATHEMATICAL CONTENT

MCCK.OA.1 Represent addition and subtraction with objects, fingers, mental i drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MCCK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MCCK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MCCK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MCCK.OA.5 Fluently add and subtract within 5

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In developing the meaning of addition and subtraction with whole numbers, students should also encounter the properties of operations, such as the commutativity and associativity of addition. Although some students discover and use properties of operations naturally, teachers can bring these properties to the forefront through class discussions (NCTM Principles and Standards, 2012).

ESSENTIAL QUESTIONS

- What happens when I decompose a quantity?
- How can I use different combinations of numbers to represent the same quantity?

- How can I represent problems using objects, pictures, and numbers?
- Why is it important that I can build the number combinations for the number 5?
- How do you know when your answer makes sense?

MATERIALS

- *The Penny Pot* by Stuart Murphy
- Recording sheet and cut out candy (if needed)

GROUPING

whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Begin by reading *Penny Pot* (or a similar book) to the class and introduce the task. In this task two sisters go to the candy store and their mother has given them a nickel to spend. The sisters spend the whole nickel and each of them buys a piece of candy. Students must determine all of the possible combinations of candy they could possibly buy. Be sure to discuss vocabulary and clarify any misconceptions students may have with the task.

Part I

Emma and Audrey went to the candy store and they had 1 nickel to spend between the two of them. If they spent the whole nickel and each got 1 item from the candy store, what are some of the items they could have bought?

FORMATIVE ASSESSMENT QUESTIONS

- Are there any more was to decompose the number 5? How do you know?
- Why did you decide to do it his way?
- Are you sure that you have found them all? Why do you think so? How do you know?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension

- As an extension to this activity the amount of money and/or candy purchased could be increased. In addition, students could find all of the possible combinations of candy that could be bought using the entire nickel. In this extension more than two items can be bought. Example: 5 suckers
  1 lollipop and 3 suckers
  2 lollipops and 1 sucker and 3 apples

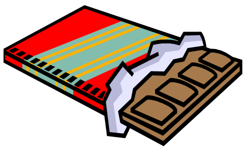
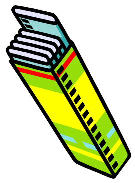
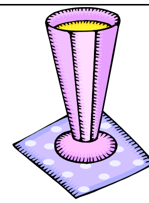
Some students may realize that they can have as many apples as they want and the amount of money spent does not change the total. The zero property can be discussed ONLY IF IT IS IDENTIFIED BY THE STUDENTS.

Intervention

- Allow students to work through the stages at a speed that is appropriate for their abilities. Some students may need additional experiences acting out problems, using manipulatives, or drawing pictures.
- Students can use cut out pictures of candy and physically place them in combination pairs that make 5.
- Students that have difficultly fixing a quantity to the candy can have cost of each item written on the picture.

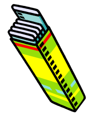
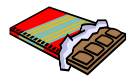
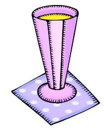
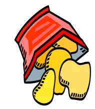
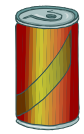
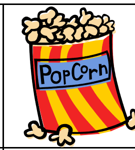
The Candy Store

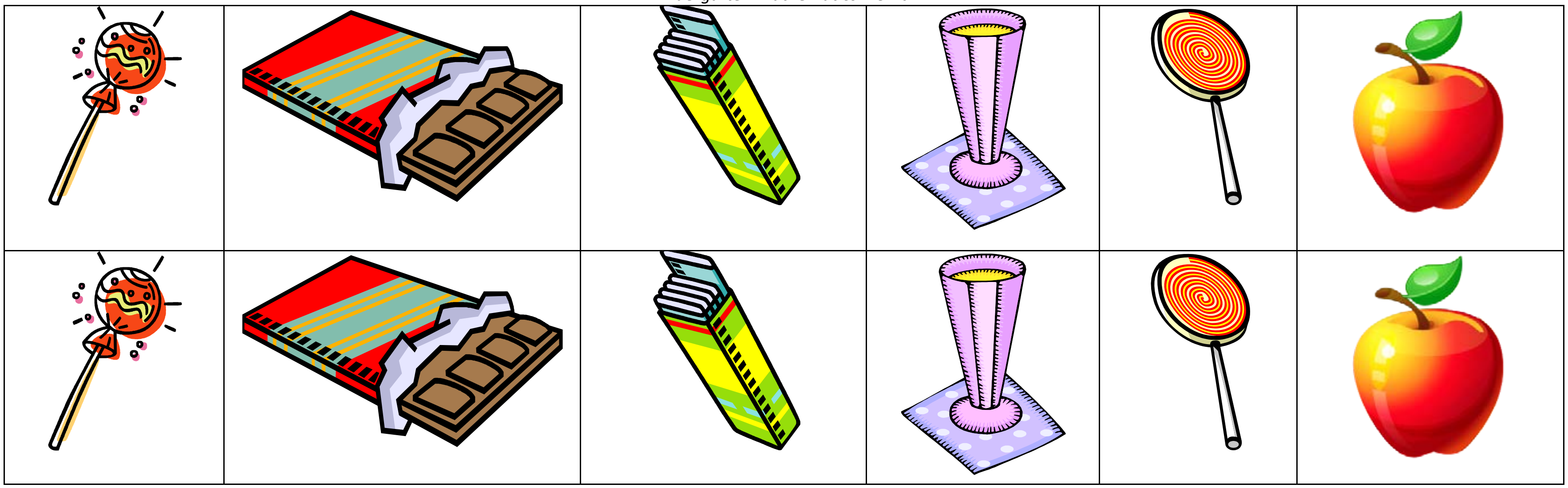
Emma and Audrey went to the candy store and they had 1 nickel to spend between the two of them. If they spent the whole nickel and each got 1 item from the candy store, what are some of the items they could have bought?

					
2 cents Lollipop	4 cents Chocolate Bar	3 cents Gum	5 cents Milkshake	1 cent Sucker	Free Apple

The Candy Store

Emma and Audrey went to the candy store and they had 2 nickels to spend between them. If they spent both nickels and each got only 1 item from the candy store, what are some of the items they could have bought?

										
Free Apple	**1 cent** sucker	**2 cents** lollipop	**3 cents** gum	**4 cents** Chocolate bar	**5 cents** milkshake	**6 cents** chips	**7 cents** pop	**8 cents** jellybeans	**9 cents** popcorn	**10 cents** Hotdog & pop



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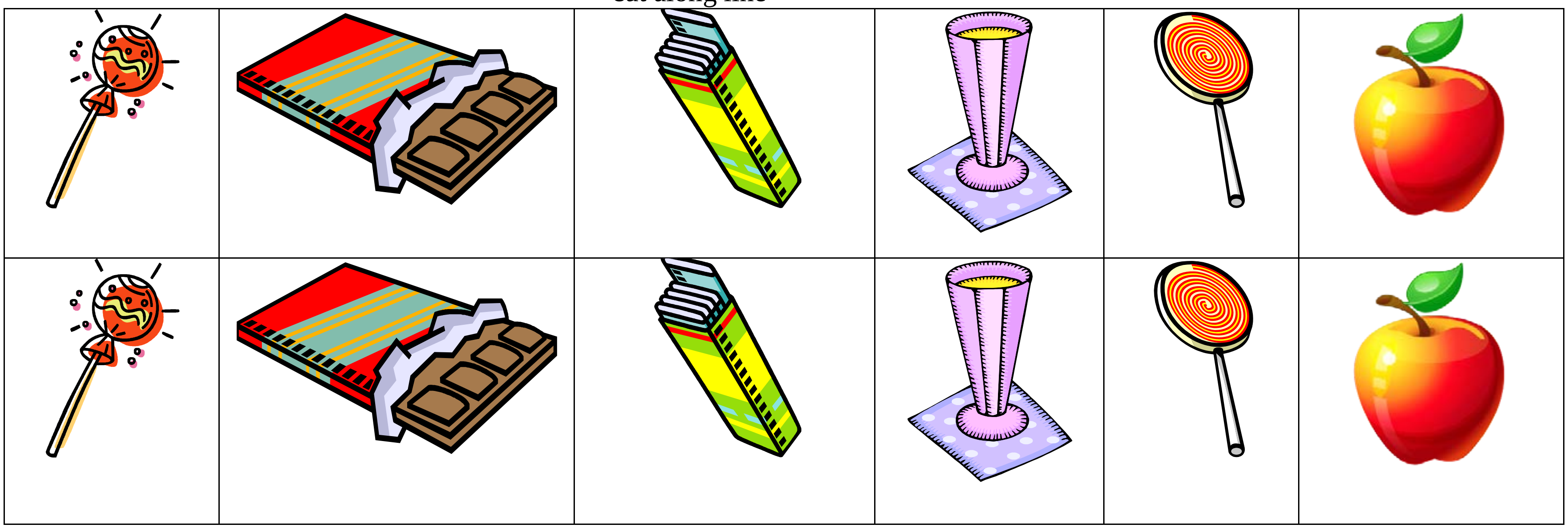


PopCorn
PopCorn